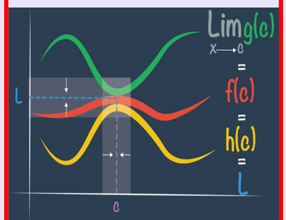
Math 261

Spring 2021

Lecture 9



Evaluate $\lim_{x\to0}\frac{\tan 5x}{\sin 3x} = \frac{\tan 0}{\sin 0} = \frac{0}{0}$ I.F.

$$\lim_{x\to0}\frac{\tan 5x}{\sin 3x} = \lim_{x\to0}\frac{\frac{\sin 5x}{\cos 5x}}{\sin 3x} = \lim_{x\to0}\frac{\sin 5x}{\cos 5x\cdot\sin 3x}$$

$$\lim_{x\to0}\frac{\sin x}{x}=1 \quad \& \quad \lim_{x\to0}\frac{1-\cos x}{x}=0$$

$$= \lim_{x\to0}\frac{1}{\cos 5x}\cdot\lim_{x\to0}\frac{\sin 5x}{\sin 3x}$$

$\lim_{x\to0}\frac{1}{\cos 5x} \to 1$

$$= \frac{1}{\cos 0}\cdot\lim_{x\to0}\frac{\frac{5\sin 5x}{5x}}{\frac{3\sin 3x}{3x}} = \frac{1}{1}\cdot\frac{5}{3}\cdot\frac{\lim_{x\to0}\frac{\sin 5x}{5x}}{\lim_{x\to0}\frac{\sin 3x}{3x}} = \frac{1}{1}\cdot\frac{5}{3}\cdot\frac{1}{1}$$

$$= \boxed{\frac{5}{3}}$$

Evaluate $\lim_{\theta\to 0}\dfrac{\theta}{\cos\theta}=\dfrac{0}{\cos 0}=\dfrac{0}{1}=\boxed{0}$

Evaluate $\lim_{x\to 0}\dfrac{1-\cos 4x}{1-\cos 3x}=\dfrac{1-\cos 0}{1-\cos 0}=\dfrac{1-1}{1-1}=\dfrac{0}{0}$ I.F.

We know $\lim_{x\to 0}\dfrac{1-\cos x}{x}=0$

$$\lim_{x\to 0}\frac{\dfrac{4(1-\cos 4x)}{4x}}{\dfrac{3(1-\cos 3x)}{3x}}=\frac{4}{3}\cdot\frac{\lim_{x\to 0}\dfrac{1-\cos 4x}{4x}}{\lim_{x\to 0}\dfrac{1-\cos 3x}{3x}}$$

$$=\frac{4}{3}\cdot\frac{0}{0}=\frac{0}{0}$$

I.F.

$$\lim_{x\to 0}\frac{1-\cos 4x}{1-\cos 3x}=\lim_{x\to 0}\frac{(1-\cos 4x)(1+\cos 4x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 4x)(1+\cos 3x)}$$

Let's try Conjugates

$$=\lim_{x\to 0}\frac{\sin^2 4x\cdot(\underbrace{1+\cos 3x}_{\to 2})}{\sin^2 3x\cdot(\underbrace{1+\cos 4x}_{\to 2})}$$

$$=\frac{2}{2}\left[\lim_{x\to 0}\frac{\sin 4x}{\sin 3x}\right]^2$$

$$=\left[\lim_{x\to 0}\frac{4\dfrac{\sin 4x}{4x}}{3\dfrac{\sin 3x}{3x}}\right]^2=\frac{16}{9}\cdot\left[\frac{\lim_{x\to 0}\dfrac{\sin 4x}{4x}}{\lim_{x\to 0}\dfrac{\sin 3x}{3x}}\right]^2=\frac{16}{9}\cdot 1$$

$$=\boxed{\frac{16}{9}}$$

Find a nonzero value for K such that

$$f(x)=\begin{cases}\dfrac{\tan Kx}{x} & x<0\\ 3x+2K^2 & x\geq 0\end{cases}$$ is Cont. at $x=0$.

To be Cont. at $x=0 \Rightarrow \lim_{x\to 0} f(x) = f(0)$

$= 2K^2$

$f(0) = 3(0)+2K^2 = 2K^2$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(3x+2K^2) = 2K^2$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\tan Kx}{x} = \lim_{x\to 0^-} \frac{K\sin Kx}{\cos Kx \cdot Kx} = K \lim_{x\to 0^-} \frac{1}{\cos Kx} \cdot \lim_{x\to 0^-} \frac{\sin Kx}{Kx}$

$= K \cdot \frac{1}{1} \cdot 1 = K$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \Rightarrow K = 2K^2$

Solve $2K^2 = K$

$2K^2 - K = 0$

$K(2K-1) = 0$

$\boxed{K=0}$ $\boxed{K=\frac{1}{2}}$

Class QZ 3

For $\varepsilon = .05$, find δ such that $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$

Verify $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$ ✓ $\frac{2^2-4}{2-2} = \frac{0}{0}$ I.F.

$\lim_{x\to 2} \frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2}(x+2) = 2+2 = 4$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$\left|\frac{x^2-4}{x-2} - 4\right| < \varepsilon$ " $|x-2| < \delta$

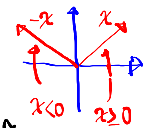
$|x+2-4| < \varepsilon$ " $|x-2| < \delta$

$|x-2| < \varepsilon$ " $|x-2| < \delta$

Pick $\delta = \varepsilon$ since $\varepsilon = .05$, Pick $\delta = .05$

For $\varepsilon = .025$, find $\delta > 0$ such that $\lim_{x \to -5}(2|\overset{-x}{x}| - 3) = 7$.

Verify $\lim_{x \to -5}(\overbrace{2|x|-3}^{f(x)}) = 2|-5| - 3 = 2 \cdot 5 - 3 = \overset{L}{7}$ ✓



$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|2|x| - 3 - 7| < \varepsilon$ whenever $|x - (-5)| < \delta$

$|2|x| - 10| < \varepsilon$ 〃 $|x + 5| < \delta$

$|2 \cdot (-x) - 10| < \varepsilon$ 〃 $|x + 5| < \delta$

$|-2x - 10| < \varepsilon$ 〃 $|x + 5| < \delta$

$|-2(x + 5)| < \varepsilon$ 〃 $|x + 5| < \delta$

$2|x + 5| < \varepsilon$

$|x + 5| < \frac{\varepsilon}{2}$

Pick $\delta = \frac{\varepsilon}{2}$

For $\varepsilon = .025$

$\delta = \frac{.025}{2}$ $\boxed{\delta = .0125}$

For $\varepsilon > 0$, find $\delta > 0$ such that $\lim_{x \to 3} x^3 = 27$. Verify $\lim_{x \to 3} x^3 = 3^3 = 27$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^3 - 27| < \varepsilon$ 〃 $|x - 3| < \delta$

$|x^3 - 3^3| < \varepsilon$ 〃 $|x - 3| < \delta$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$|(x - 3)(x^2 + 3x + 9)| < \varepsilon$ 〃 $|x - 3| < \delta$

$|x^2 + 3x + 9|\,|x - 3| < \varepsilon$

δ not to be more than 1

$|x - 3| < 1$

Keep

$|x - 3| < \frac{\varepsilon}{|x^2 + 3x + 9|}$

$-1 < x - 3 < 1$

$2 < x < 4$

Bound

$x^2 < 16$

$3x < 12$

Maximize deno to minimize the fraction.

$x^2 + 3x + 9 < 16 + 12 + 9 = 37$

$\boxed{\text{So } \delta = \min\left\{1, \frac{\varepsilon}{37}\right\}}$

Evaluate $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ for $f(x) = \sqrt[3]{x}$

$$\lim_{h \to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} = \frac{0}{0} \text{ I.F.}$$

we need to rationalize the number.

Conjugate needed.

$$= \lim_{h \to 0} \frac{(\sqrt[3]{x+h} - \sqrt[3]{x})(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2})}{h(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2})}$$

Recall $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$$= \lim_{h \to 0} \frac{(\sqrt[3]{x+h})^3 - (\sqrt[3]{x})^3}{h(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2})}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h}^{\,1} - \cancel{x}}{\cancel{h}(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2})}$$

$$= \frac{1}{\sqrt[3]{x^2} + \sqrt[3]{x}\sqrt[3]{x} + \sqrt[3]{x^2}} = \frac{1}{3\sqrt[3]{x^2}}$$

$$= \frac{\sqrt[3]{x}}{3x} \checkmark$$

$\boxed{\text{SG4 Q8 } \sqrt[5]{x}}$

$\lim\limits_{x \to \infty}$, $\lim\limits_{x \to -\infty}$